

MATH-422 Advanced Group Theory

Credit Hours: 3-0

Prerequisite: MATH-324 Abstract Algebra

Course Objectives: This course covers some advanced topics in group theory such as internal and external direct products of groups, classification of finitely generated abelian groups, commutator subgroup, group action on a set, conjugate elements, conjugacy classes, Sylow's theorems, simple groups, free groups, automorphisms of groups.

Core Contents: Direct product of groups, group action on a set, conjugate elements, conjugacy classes, Cauchy's theorem for abelian groups, Sylow's theorems, free groups, automorphisms of groups.

Detailed Course Contents: Direct product of groups, internal direct product, external direct product, computation in factor group, the correspondence theorem, commutator subgroup, simple groups, group action on a set, conjugate subgroups, conjugate elements and conjugacy classes, the class equation of a group, p-groups, Sylow p-subgroups, Cauchy theorem for abelian groups, Sylow's first theorem, Sylow's 2nd and 3rd theorems, applications of Sylow's theorems, automorphisms, group of automorphisms, inner automorphisms, group of automorphisms of a cyclic group, generators and relations, free groups.

Course Outcomes: On successful completion of this course, students will know

- Direct product of groups, external direct product, internal direct product
- The classification of finitely generated abelian groups
- The correspondence theorem for groups
- Commutator subgroup
- Conjugate subgroups, conjugate elements and conjugacy classes
- Group action on a set
- The class equation of a group
- p-groups, Sylow p-subgroups, Cauchy's theorem for abelian groups
- Sylow's theorems, simple groups, applications of Sylow's theorems
- Automorphisms, group of automorphisms, Inner automorphisms,

- Group of automorphisms of a cyclic group
- Free groups.

Textbooks:

1. J. B. Fraleigh, A first course in abstract algebra (7th Ed.), 1998, Addison-Wesley publishing. (referred as J.F)
2. I.N. Herstein, Topics in Algebra (2nd Ed.), New York, John Wiley & Sons, Inc., 1975.(referred as I.H)

Reference Books:

1. W. K. Nicholson, Introduction to Abstract Algebra, (3rd Ed), 2007, John Wiley & Sons.
2. J. A. Gallian, Contemporary Abstract Algebra. (7th Ed.) Cole, Belmont, CA, 2009.

Weekly Breakdown		
Week	Section	Topics
1	J.F Sec.11	Direct product of groups, internal direct product, examples and related results.
2	J.F Sec.11	External direct product of groups, examples and related results.
3	J.F Sec. 11	Classification of finitely generated abelian groups.
4	J.F Sec.15	Factor-group computation, the correspondence theorem, applications of the correspondence theorem.
5	J.F Sec.15	Simple groups, commutator subgroups, examples and related results.
6	J.F Sec. 16	Group action on a set, examples and related results.
7	J.F Sec.36	Conjugate elements and conjugacy classes, the class equation of a group.
8	J.F Sec 36	p-Groups, Sylow p-subgroups, definitions, examples and related results.

9	Mid Semester Exam	
10	J.F Sec.36	Cauchy theorem for abelian groups, Sylow's first theorem.
11	J.F Sec.36	Sylow's 2nd and 3rd theorems.
12	J.F Sec. 37	Applications of Sylow's theorems to simple groups.
13	J.F Sec.38, 39	Free abelian groups, proof of the fundamental theorem for finitely generated abelian group.
14	J.F Sec.39	Free groups, group presentations.
15	I.H Sec. 2.8	Automorphisms, group of automorphisms definitions and examples and related results.
16	I.H Sec. 2.8	Inner automorphisms, group of automorphisms of a cyclic group, and related results.
17		Review
18	End Semester Exam	